# AN EPIDEMIC OF ZOMBIE INFECTION: A MATHEMATICAL MODEL

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## ABSTRACT

Zombies are a popular figure in pop culture/entertainment and they are usually portrayed as being brought about through an epidemic. Consequently we model a zombie attack, using biological assumptions based on popular zombie movies. In particular, the behavior of zombie attack over time is modeled using a deterministic Susceptible, Zombie, and Removed (SZR) model. The dynamics such as equilibrium, stability and sensitivity of the model with respect to changes in model parameters; are determined and the outcome is illustrated with numerical solutions.


**Keywords:** Zombies, model, doomsday, epidemic, Eulerõs method, population.

## INTRODUCTION

A zombie is a reanimated human corpse that feeds on living human flesh [1]. Stories about zombies are derived from the Afro-Caribbean spiritual belief system of Vodou (anglicized voodoo). These stories regarded people as being influenced by a powerful sorcerer. The walking dead became widely known in the modern horror fiction solely as a result of George A. Romeroõs 1968 film, Night of the Living Dead [2]. Many etymologies of the word ózombieô exist. According to the Merriam-Webster dictionary, the word ózombieô comes from the word ózombiõ, used in the Louisiana Creole or the Haitian Creole. Creole culture has it that, a zonbi represents a person who died and was then brought to life without speech or free will.

It is the belief of the followers of Vodou a dead person can be revived by a sorcerer. After revival, the zombies remain under the control of the sorcerer for they have no will of their own. Zombie is also another name for a Voodoo snake god. The idea of the Zombie is also contained in several other cultures as China, Japan, the Pacific, India, Persia, the Arabs and the America.

Contemporary zombies, such as the ones described in books, films and games are quite different from the Voodoo and the folklore zombies. The standard set in the movie Night of the Living Dead is followed by modern zombies. The ghouls are portrayed as being mindless monsters who do not feel pain and who have an immense appetite for human flesh. Their interest is to kill, eat or infect people. The undead move in small, irregular steps and show signs of physical decomposition such as rotten flesh, discoloured eyes and open wounds. Modern zombies are often compared to an apocalypse, where civilization could collapse due to a plague of the undead. The background stories of zombie movies, video games are purposefully vague and inconsistent in explaining the origin of zombies. Some ideas include radiation (Night of the Living Dead [2]), exposure to air-borne viruses, mutated diseases carried by various vectors.

When a Susceptible individual gets bitten by a zombie, it creates an open wound. The resultant wound has the zombieȏs saliva in and around it. This bodily fluid mixes with the blood, thus infecting the (previously susceptible) individual.

This paper presents a basic zombie model which is best characterized by the popular-culture zombie. The zombies in question are of the classical pop-culture zombies who are slow moving, cannibalistic and undead.

## 2 THE BASIC MODEL

The basic model consists of three basic classes:-

- Susceptible (S).
- Zombie (Z).
- Removed (R).

The Susceptible class consists of individuals that are not yet infected. Susceptible can die a natural death, that is, non-zombie related death (parameter $\delta$). The removed class consists of individuals who have died either a natural death or through a zombie infection. In this class, humans can resurrect and become a zombie irrespective of the cause of death ( parameter $\zeta$).

Susceptible can become zombies through transmission via a contact with a zombie (transmission parameter β). Therefore, the two only possible sources of new zombies are:-

(1) The resurrected from the newly deceased (removed group).
(2) Susceptibles who have been infected through a bite from a zombie.

It is assumed that the birth rate is constant, **π**. In addition, zombies join the removed class upon being ódefeatedô. This is possible by removing the head or destroying the brain of the zombie ( parameter α). We also assume that zombies neither infect nor defeat other zombies.

Hence, the basic model is described by the following ordinary differential equations (ODEs):

dS/dt = ´ ï ϐSZ ï ŭS

dZ/dt = ϐSZ + ϛR ï ŬR

dR/dt = ŭS + ŬSZ ï ϛR

This is illustrated in Figure 1.

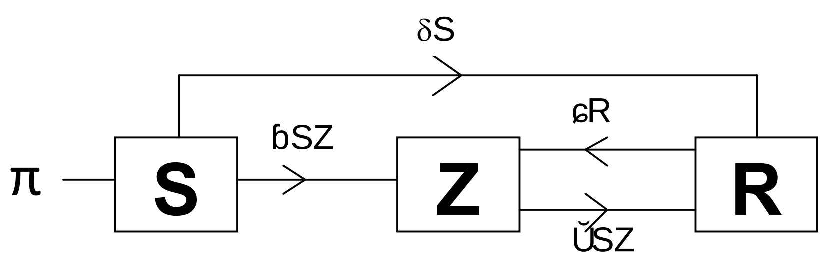


Figure 1: The basic model.

This model is slightly different from the basic **SIR** models that usually describe infectious diseases in the sense that it has two mass-action transmissions, and consequently more than one non-linear term. As such this model is slightly more complex than the basic SIR models. Mass-action incidence specifies that an average member of the population makes contact sufficient to transmit infection with ϐN others per unit time, where N is the total population without infection. In this case, the infection is zombification. The probability that a random contact by a zombie is made with a susceptible is S/N; thus the number of new zombies through this transmission process in unit time per zombie is:

(бN)(S/N)Z = бSZ

We also assume that a susceptible can defeat a zombie during their contact and thus avoid zombification, and each susceptible is capable of resisting zombification at a rate Ŭ. So, employing the same idea as above with the probability Z/N of random contact of a susceptible with a zombie (not the probability of a zombie attacking a susceptible) the number of zombies destroyed through this process per unit time per susceptible is:-

(ŬN)(Z/N)S = ŬSZ

The above derived ODEs satisfy

dS/dt + dZ/dt + dR/dt = ´

i.e. S + Z + R Ÿ ∞

as t Ÿ ∞, if ´ ≠ 0. Clearly, S Ÿ ∞, thus, this leads to a ódoomsdayô scenario, an outbreak of zombies will result in collapse of civilization, as large numbers of people are either zombified or dead.

Let us assume that the outbreak happens over a short timescale such that we can ignore birth and background death rates. i.e. ´ = ŭ = 0.

Plugging in ´ = ŭ = 0 and setting the ODEs to 0, we have

-бSZ = 0.

$$\beta SZ + \zeta R - \alpha SZ = 0.$$

$$\alpha SZ - \zeta R = 0.$$

From the first equation, we have either S = 0 or Z = 0.

When S = 0, we get the "doomsday" equilibrium,

$$(\bar{S}, \bar{Z}, \bar{R}) = (0, Z, 0)$$

When Z = 0, we have the disease-free equilibrium.

$$(\bar{S}, \bar{Z}, \bar{R}) = (N, 0, 0)$$

These equilibrium points reveal that regardless of their stability, human-zombie coexistence is impossible.

## 3 NUMERICAL RESULTS

In the following figures, the curves show the interaction between susceptibles and zombies over a period of time. Euler's method was used to solve the ODE's and MATLAB was used to code the equations and to plot the graphs. The graphs depict the sensitivities of the model with respect to changes in the model parameters ($\alpha$, $\beta$, $\delta$, $\zeta$).

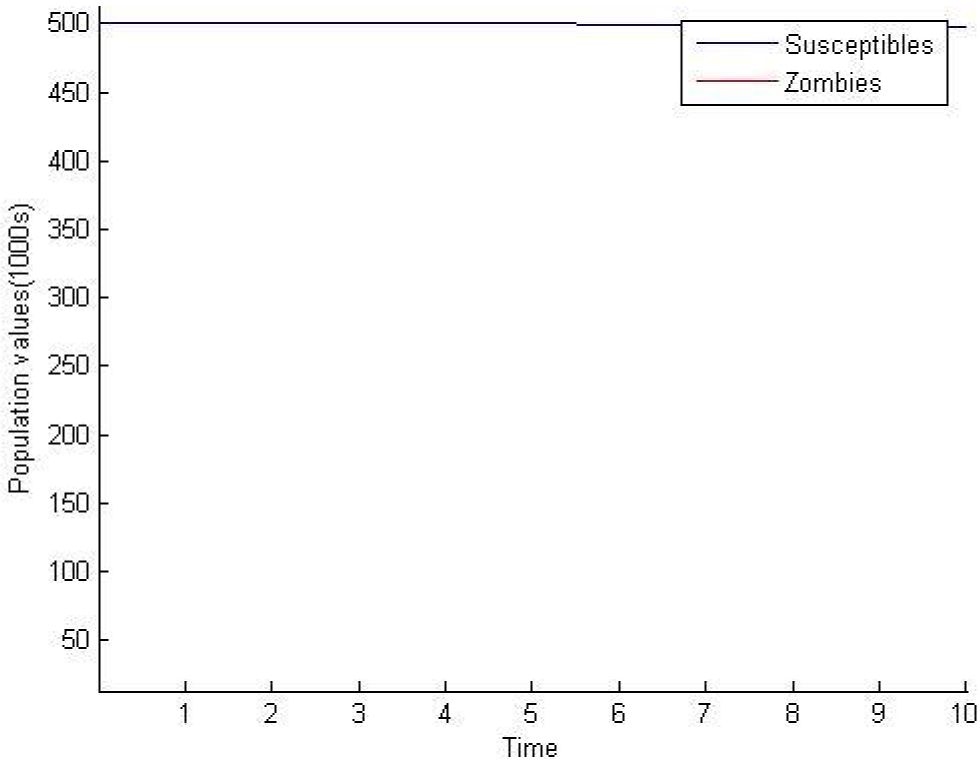


Figure1. The case of no zombies

This graph shows us the case of no zombies ï disease free equilibrium. In this case, the number of susceptibles is equal to the total population for all values of t while there are no zombies for all t values.

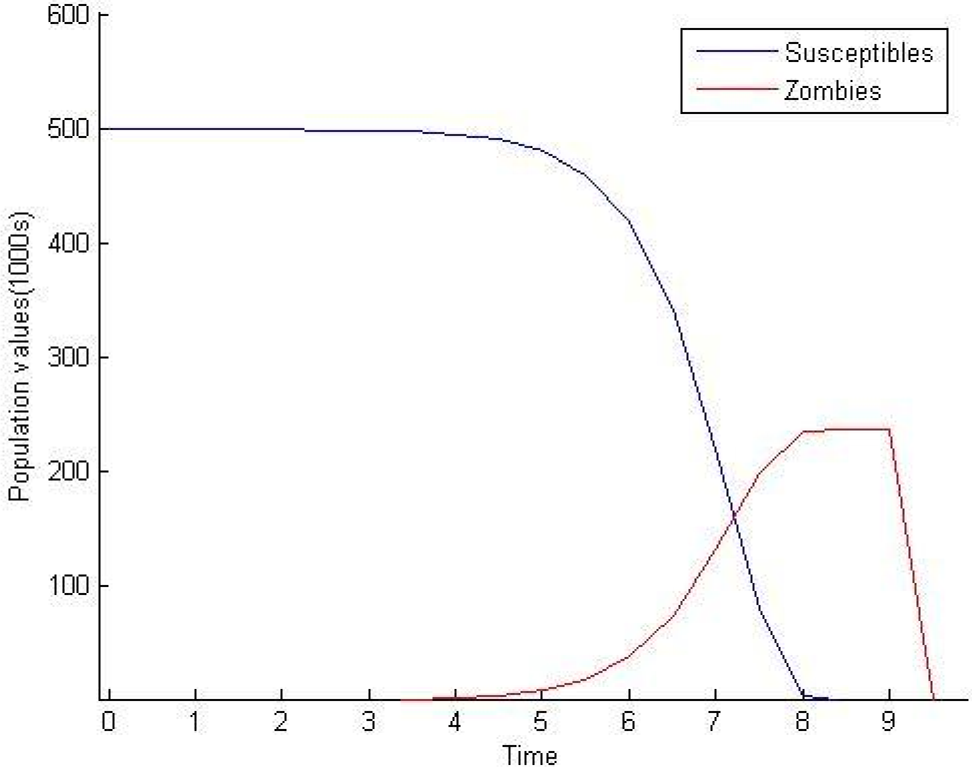


Figure2: When Background death (ŭ) is increased from 0.0001 to 0.5

At this stage, the number of zombies increases as soon as the number of susceptibles starts falling. Note that the rate at which susceptibles fall is rapid, this is because the rate at which humans move to the removed class have been increased, giving the zombies an edge over the susceptibles thereby making it very easy for dead humans to be resurrected to become zombies.

We can also notice that at 8 to 9 hours, the number of zombies remains constant. This is because the population of susceptibles has been overwhelmed by zombies leaving zombies no human to infect and therefore their population drops periodically as they have no human to feed on to survive.

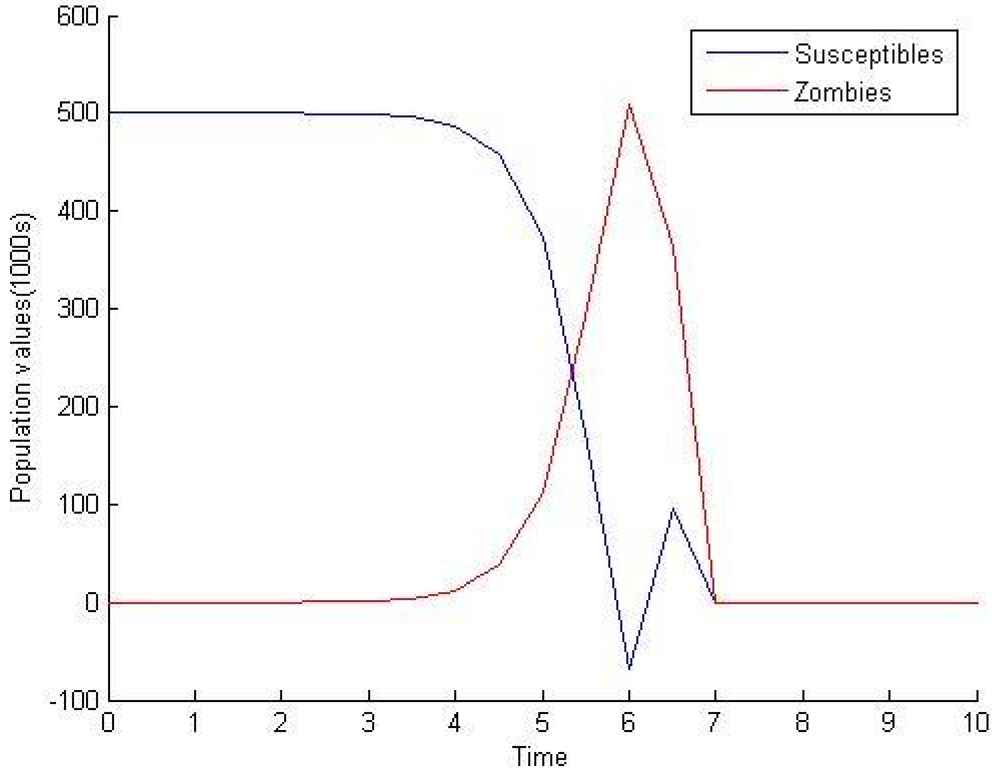


Figure3: When (Ŭ) is decreased and background death rate (ŭ) is increased.

In this graph, Ŭ was decreased from 0.005 to 0.001 and ŭ was increased from 0.0001 to 0.5. We can see that when the number of susceptibles decreases after 4 hours, the number of zombies increases at an equal rate, this is due to the fact that the resistance rate (Ŭ) is low and the death rate is high causing an increase in zombies as the dead can rise and become a zombie, as we earlier stated.

At 6 hours, we can see that zombies take over the population and susceptibles fall to the smallest minimum and finally at 7 hours we have no susceptible in the population. This is the effect of low resistance which makes a susceptible individual vulnerable to a zombie attack and increase in background death rate which in turn, increases the number of zombies in the population as dead humans are resurrected.

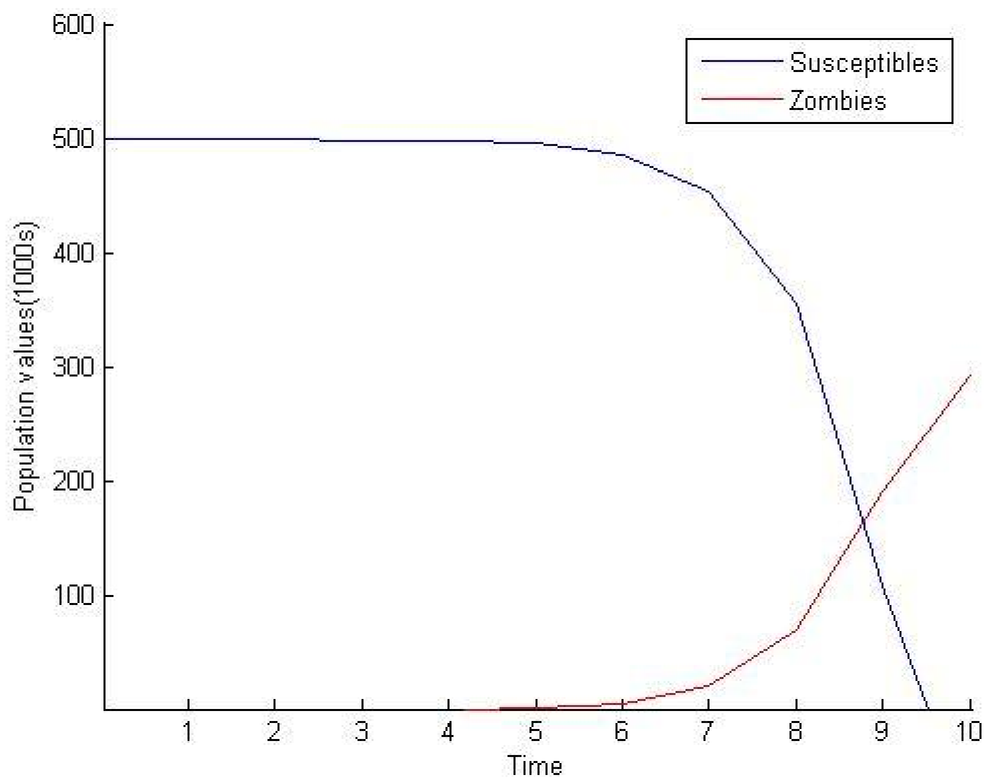


Figure4: The contact rate (ɓ) increased, background death rate (ŭ) increased.

In this case, ɓ was increased from 0.0095 to 0.0098 and the death rate increased. As we can notice from the graph that at 5 hours, the number of zombies starts increasing and the number of susceptibles starts reducing per hour basis this is as a result of the increase in contact rate (ɓ) and an increase in the death rate (ŭ) which makes the number of zombies increase as most susceptibles are subdued. Before the maximum time is reached, zombies have taken over the whole population. From the graph, we can also see that between 9 to 10 hours, the number of susceptibles falls to 0 and the number of zombies increases to 300,000 not 500,000, which was the initial number of susceptibles in the population. This could be as result of the increase in death rate where by a susceptible can die a natural death without being zombified and are not yet resurrected to become a zombie.

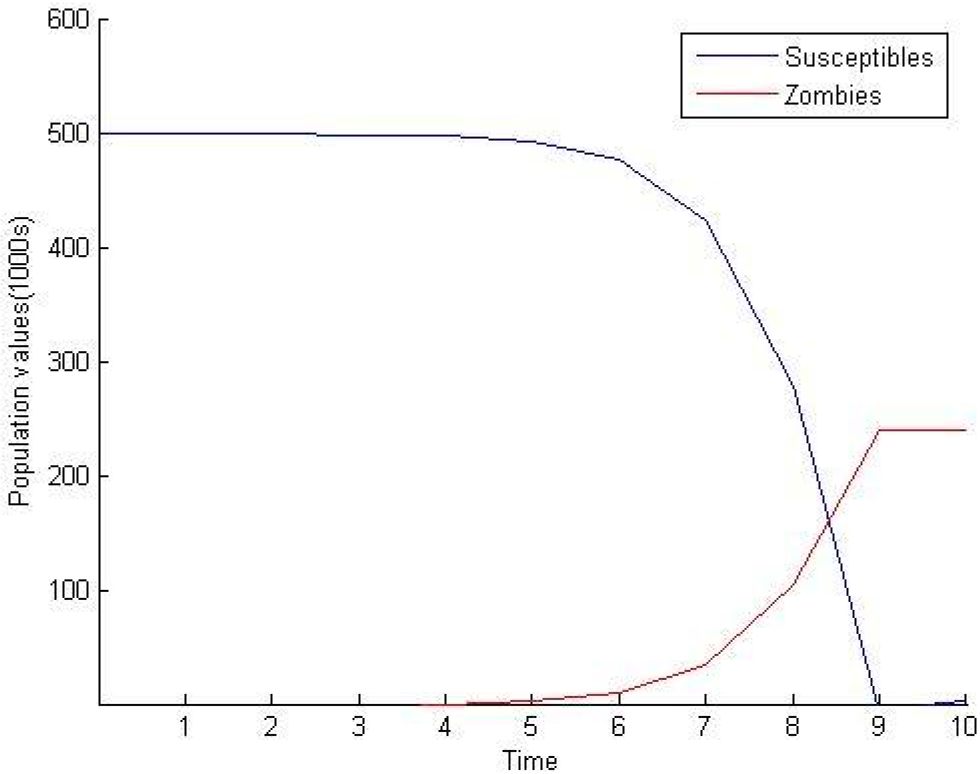


Figure5: The resurrection rate (ϵ) increased at the same level with the death rate (ŭ).

Here, the resurrection parameter was increased at the same level with the background death rate, this is because the level at which dead humans are resurrected is as a result of the death that occurs. Therefore, we can see that the rate at which susceptibles fall is more rapid than that of the zombies. For example, between 7 to 8 hours, susceptibles has fallen from about 420,000 to 320.000, that is, 100,000 susceptibles have died or are infected. At the same time, the number of zombies increases from about 20,000 to 100,000, bringing us to an increase of 80,000 in the number of zombies. Between 8 to 9 hours, all susceptibles have been infected and their population falls from 300,000 to 0 and the number of zombies increases from a 100,000 to 250,000. At 9 to 10 hours, the number of susceptibles remains constant at 0 and the number of zombies remains constant at 250,000.

## 4 CONCLUSION AND FUTHER WORK

The results presented in this paper assumed that the timescale of the outbreak was short so that natural birth and death rates could be ignored. If the timescale of the outbreak increases then the results is the doomsday scenario. The key difference between the model presented here and other models of infectious disease is that the dead can resurrect. Obviously, this is an unrealistic scenario if taken literally. However, possible real life applications may include allegiance to political parties, or diseases with a dominant infection. An outbreak of zombie infection is likely to be catastrophic if aggressive control measures are not taken against the undead.

Our future work will center on building effective control measures such quarantine, treatment and strategic eradication into the model. This will allow us to investigate the most effective way to contain the epidemic.

## REFERENCES:


[1] Brooks, Max, 2003 The Zombie Survival Guide - Complete Protection from the Living Dead, Three Rivers Press, pp. 2-23

[2] Romero, George A. (writer, director), 1968 Night of the Living Dead.

[3] Brauer, F. Compartmental Models in Epidemiology. In: Brauer, F., van den Driessche,P., Wu, J. (eds). Mathematical Epidemiology. Springer Berlin 2008.

[4] Diekmann O. and Heesterbeek J. A. P (2000). Mathematical epidemiology of infectious diseases: model building, analysis and interpretation. Chichester: Wiley. [The modern theory of epidemic models presented with numerous mathematical exercises and their solution.]

[4] Capcom, Shinji Mikami (creator), 1996-2007 Resident Evil.

[5] Capcom, Keiji Inafune (creator), 2006 Dead Rising

[6] Pegg, Simon (writer, creator, actor), 2002 Shaun of the Dead.

[7] Boyle, Danny (director), 2003 28 Days Later.

[8] Snyder, Zack (director), 2004 Dawn of the Dead.

[9] Brauer, F. Compartmental Models in Epidemiology. In: Brauer, F., van den Driessche,P., Wu, J. (eds). Mathematical Epidemiology. Springer Berlin 2008.

[10] Brooks, Max, 2006 World War Z - An Oral History of the Zombie War, Three Rivers Press

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